Name: ______________________________

Instructor: __________________________

Math 10560, Practice Exam 2.
March 20, 2024

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 14 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice	________
11.	________
12.	________
13.	________
14.	________
15.	________
16.	________
Total	________

Name: ____________________________

Instructor: _________________________

Multiple Choice

1.(6 pts.) Evaluate the improper integral

$$\int_4^\infty \frac{1}{(x-2)(x-3)}\,dx.$$

(a) $\ln 3$ (b) $\ln \frac{1}{2}$ (c) $\ln 2$

(d) the integral diverges (e) $3\ln 2$

2.(6 pts.) What can be said about the integrals

$$(i)\ \int_0^1 \frac{e^x}{x^2}\,dx;$$

$$(ii)\ \int_1^\infty \frac{\cos^2 x}{x^2}\,dx?$$

(a) both (i) and (ii) converge

(b) both (i) and (ii) diverge

(c) (i) converges and (ii) diverges

(d) (i) diverges and (ii) converges

(e) neither integral (i) nor (ii) is improper

Name: ____________________________

Instructor: __________________________

3.(6 pts.) Which of the following is an expression for the arclength of the curve $y = \cos x$ between $x = \frac{-\pi}{2}$ and $x = \frac{\pi}{2}$?

(a) $2\int_0^{\frac{\pi}{2}} \sqrt{1 + 2\sin^2 x}\, dx.$

(b) $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sqrt{1 - \sin^2 x}\, dx.$

(c) $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sqrt{1 + \sin^2 x}\, dx.$

(d) $\frac{\pi^2}{2}$

(e) $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sqrt{1 - \cos^2 x}\, dx.$

4.(6 pts.) Consider the following sequences:

$$(I)\ \left\{(-1)^n \frac{n^2-1}{2^n}\right\}_{n=1}^{\infty} \qquad (II)\ \left\{(-1)^n \frac{n^2-1}{2n^2}\right\}_{n=1}^{\infty} \qquad (III)\ \left\{(-1)^n n \ln(n)\right\}_{n=1}^{\infty}$$

Which of the following statements is true?

(a) Sequences I and II converge but sequence III diverges.

(b) All three sequences converge.

(c) Sequences II and III converge but sequence I diverges.

(d) All three sequences diverge.

(e) Sequence I converges but sequences II and III diverge.

Name: ______________________________

Instructor: __________________________

5.(6 pts.) Find the sum of the following series:

$$\sum_{n=1}^{\infty} \frac{(-1)^n 2^{n+1}}{3^n}.$$

(a) This series diverges.

(b) $-\frac{4}{5}$

(c) $-\frac{3}{5}$

(d) $\frac{4}{5}$

(e) $\frac{3}{5}$

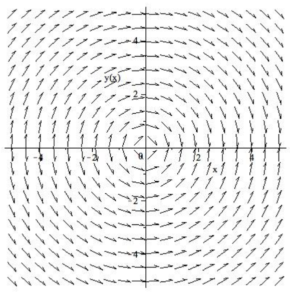
Name: ____________________________

Instructor: ________________________

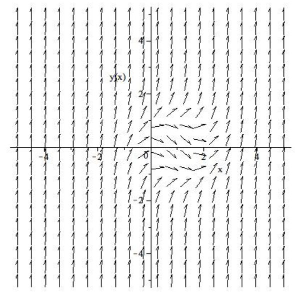
6.(6 pts.) Which of the following gives the direction field for the differential equation

$$y' = y^2 - x^2$$

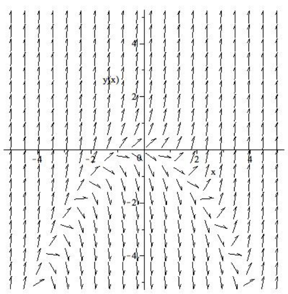
Note the letter corresponding to each graph is at the lower left of the graph.



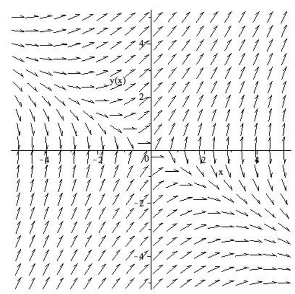
(a)



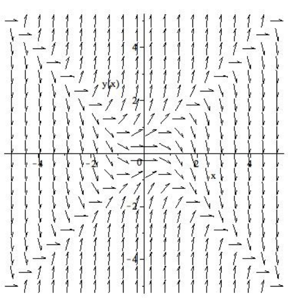
(b)



(c)



(d)



(e)

Name: ______________________________

Instructor: __________________________

7.(6 pts.) Use Euler's method with step size 0.1 to estimate $y(1.2)$ where $y(x)$ is the solution to the initial value problem

$$y' = xy + 1 \qquad y(1) = 0.$$

(a) $y(1.2) \approx .112$

(b) $y(1.2) \approx .211$

(c) $y(1.2) \approx .101$

(d) $y(1.2) \approx .201$

(e) $y(1.2) \approx .111$

8.(6 pts.) Find the solution of the differential equation

$$\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{1+x^2}$$

with initial condition $y(0) = 0$.

(a) $y = \dfrac{x}{1+x}$

(b) $y = \dfrac{1}{\sqrt{1+x^2}}$

(c) $y = \dfrac{x}{\sqrt{1+x^2}}$

(d) $y = \dfrac{x}{1+x^2}$

(e) $y = \dfrac{x^2}{\sqrt{1+x^2}}$

Name: ____________________________

Instructor: __________________________

9.(6 pts.) Find a general solution, valid for $-\frac{\pi}{2} < x < \frac{\pi}{2}$, of the differential equation

$$\frac{dy}{dx} - (\tan x)y = 1.$$

(a) $y = \dfrac{x + \sin x + C}{\cos x}$ (b) $y = \dfrac{x + \sin x + C}{\sin x}$ (c) $y = \dfrac{\sin x + C}{\cos x}$

(d) $y = \tan x + \cos x + C$ (e) $y = \dfrac{\cos x + C}{\sin x}$

10.(6 pts.) A tank contains 1000 liters of water. Brine that contains 0.5 kg of salt per liter of water is added at a rate of 5 liters per minute. The solution is kept thoroughly mixed and drains from the tank at a rate of 5 liters per minute. What's the amount of salt after 3 hours and twenty minutes?

(a) $500(1 - e)$ (b) $500(1 - \dfrac{2}{e^3})$ (c) 500

(d) $500(1 - \dfrac{1}{e})$ (e) $500(1 - \dfrac{1}{e^2})$

Name: ____________________________

Instructor: ________________________

Partial Credit

You must show your work on the partial credit problems to receive credit!

11.(10 pts.) Calculate the arc length of the curve if $y = \dfrac{x^2}{4} - \ln(\sqrt{x})$, where $2 \le x \le 4$.

Name: ______________________________

Instructor: __________________________

12.(10 pts.) (a) Circle the letter below alongside the trapezoidal approximation to

$$\ln 3 = \int_1^3 \frac{1}{x}dx \quad \text{using} \quad n = 8$$

A $\displaystyle\int_1^3 \frac{1}{x}dx \approx \frac{1}{8}\Big[1+2\Big(\frac{4}{5}\Big)+2\Big(\frac{2}{3}\Big)+2\Big(\frac{4}{7}\Big)+2\Big(\frac{1}{2}\Big)+2\Big(\frac{4}{9}\Big)+2\Big(\frac{2}{5}\Big)+2\Big(\frac{4}{11}\Big)+\Big(\frac{1}{3}\Big)\Big]$

B $\displaystyle\int_1^3 \frac{1}{x}dx \approx \frac{1}{12}\Big[1+4\Big(\frac{4}{5}\Big)+2\Big(\frac{2}{3}\Big)+4\Big(\frac{4}{7}\Big)+2\Big(\frac{1}{2}\Big)+4\Big(\frac{4}{9}\Big)+2\Big(\frac{2}{5}\Big)+4\Big(\frac{4}{11}\Big)+\Big(\frac{1}{3}\Big)\Big]$

C $\displaystyle\int_1^3 \frac{1}{x}dx \approx \frac{1}{8}\Big[1+\Big(\frac{4}{5}\Big)+\Big(\frac{2}{3}\Big)+\Big(\frac{4}{7}\Big)+\Big(\frac{1}{2}\Big)+\Big(\frac{4}{9}\Big)+\Big(\frac{2}{5}\Big)+\Big(\frac{4}{11}\Big)+\Big(\frac{1}{3}\Big)\Big]$

(b) Recall that the error E_T in the trapezoidal rule for approximating $\int_a^b f(x)dx$ satisfies

$$\Big|\int_a^b f(x)dx - T_n\Big| = |E_T| \leq \frac{K(b-a)^3}{12n^2}$$

whenever $|f''(x)| \leq K$ for all $a \leq x \leq b$.

<u>Use the above error bound to determine a value of n</u> for which the trapezoidal approximation to $\ln 3 = \int_1^3 \frac{1}{x}dx$ has an error

$$|E_T| \leq \frac{1}{3}10^{-4}.$$

Name: ______________________________

Instructor: __________________________

13.(10 pts.) Find the family of orthogonal trajectories to the family of curves given by

$$y = kx^2.$$

Name: ___________________________

Instructor: _______________________

14.(10 pts.) (10.3) A tank contains 5,000 liters of brine with 10 kg of dissolved salt. Pure water enters the tank at a rate of 50 L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. Let $y(t)$ denote the amount of salt in the tank after t minutes.

Find a formula for $y(t)$.

Name: ____________________________
Instructor: _________________________

15.(10 pts.) **Extra Problem for Practice**
Solve the initial value problem

$$xy' + xy + y = e^{-x}$$
$$y(1) = \frac{2}{e}.$$

Name: ______________________________

Instructor: __________________________

16.(10 pts.) **Extra Problem for Practice**

Solve the initial value problem $\begin{cases} x^2y' + 2xy = 1, \\ y(1) = 2. \end{cases}$

Name: ____________________________

Instructor: ________________________

The following is the list of useful trigonometric formulas:

Note: $\sin^{-1} x$ and $\arcsin(x)$ are different names for the same function and $\tan^{-1} x$ and $\arctan(x)$ are different names for the same function.

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin 2x = 2 \sin x \cos x$$

$$\sin x \cos y = \frac{1}{2}\big(\sin(x-y) + \sin(x+y)\big)$$

$$\sin x \sin y = \frac{1}{2}\big(\cos(x-y) - \cos(x+y)\big)$$

$$\cos x \cos y = \frac{1}{2}\big(\cos(x-y) + \cos(x+y)\big)$$

$$\int \sec\theta = \ln|\sec\theta + \tan\theta| + C$$

$$\int \csc\theta = \ln|\csc\theta - \cot\theta| + C$$

$$\csc\theta = \frac{1}{\sin\theta}, \quad \cot\theta = \frac{1}{\tan\theta}$$